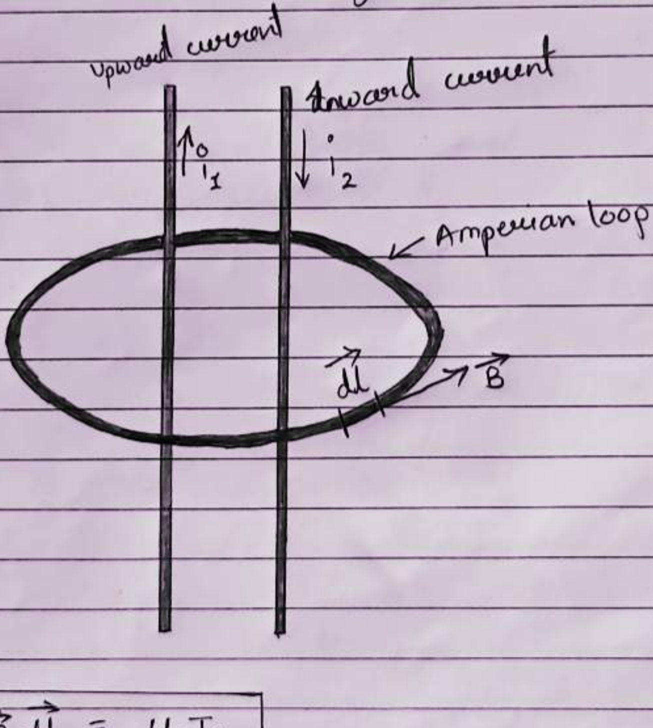
ues 1 State Ampere's law & Explain the concept of displacement current.

Ans :-

Ampere's circuital law states that line integral of steady magnetic field over a closed loop is equal to μ_0 times the total current (I) passing through the surface bounded by the loop i.e.



$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I_e}$$

I_e = Enclosed current.

The relation above involves a sign convention, given by right hand rule. let the

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Need for Displacement current

Ampere's circuital law for conduction of current during charging of a capacitor was found inconsistent. Therefore, maxwell modified Ampere's circuital law by introducing the concept of displacement current

Displacement current

Displacement current is the current that is produced by the rate of change of the electric displacement field. It differs from the normal current that is produced by the motion of the electric charge, displacement current is the quantity explained in Maxwell's equation, it is measured in Ampere. Displacement current are produced by a time varying electric field rather than moving charges.

Defination.
A physical quantity related to Maxwell's equation that has the property of the electric current is called the displacement current. displacement current is defined as the rate of change of the electric displacement (D).

Current in Capacitor
A charging capacitor has no conduction of charge accumulation in the capacitor changes

Fingers of the right hand be curled in the sense the boundary is traversed in the loop integral $\oint \vec{B}.\vec{dl}$. Then the direction of the thumb gives the sense in which the current I is regarded as positive. In the diagram shown i_1 is taken positive and i_2 is negative.

It should be similar to the gauss's law ampere's circuital law holds for any loop but it may not always facilitate an evaluation of the magnetic field in every case. It is best suited in which integral on the left side of the eqn is solved easily using the symmetry of the situation.

For several applications, It is possible to choose the loop (called an Amperian loop) such that at each point on the loops, either

(i) $\vec{B}$ is tangential to the loop and has a non zero constant magnitude B, or

(ii) $\vec{B}$ is normal to the loop or

(iii) B vanishes everywhere on the loop.

the electric field link with the capacitor that in turn produces the current called the displacement current.

$$\boxed{I_D = J_D S = S\frac{dD}{dt}}$$

where

S = area of the capacitor plate
I_D = Displacement current
J_D = Displacement current density
D = Electric field E.

$$\boxed{D = \varepsilon E}$$

ε = Permittivity of material between plates.

Displacement Current equations

Maxwell's eq^n defines the displacement current which has the same unit as the electric current. The Maxwell field eq^n is represented as

$$\nabla \times H = J + J_D$$

H = magnetic field B as $B = \mu H$
μ = permeability of material b/w the plates
J = Conducting current density
J_D = displacement current density.

we know that

$$\bar{\nabla}\cdot(\bar{\nabla}\times H) = 0$$
$$\bar{\nabla}\cdot J = -\frac{d\rho}{dt}$$

$$\bar{\nabla}\cdot J = -\bar{\nabla}\cdot \frac{dD}{dt}$$

using gauss's Law
$$\nabla\cdot D = \rho$$

ρ = electric charge density

Thus ~~electric~~ displacement current density eq^n is

$$\boxed{J_D = \frac{dD}{dt}}$$

Characteristics of displacement current

In an electric circuit there are two types of current that are conduction current and the other is displacement current. Various characteristics of displacement current are mentioned below.

→ Displacement current does not appear from the actual movement of the electric charge as in the case of the conduction current but is produced by time changing electric field.
→ Displacement current is a vector quantity.
→ Electromagnetic waves propagate with the help of displacement current.

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es 2 State and prove maxwell's equations.

Ans :- Maxwell first equation.

Maxwell's first equation is based on the Gauss law of electrostatic which states that "when a closed surface integral of electric flux density is always equal to charge enclosed over that surface."

Mathematically gauss law can be expressed as,

$$\oint \vec{D} \cdot d\vec{s} = Q_{enclosed} \quad — (1)$$

Any closed system will have multiple surfaces but a single volume, thus, the above surface integral can be converted into a volume integral by taking the divergence of the same vector, Thus mathematically it is

$$\oint \vec{D} \cdot d\vec{s} = \int \nabla \cdot \vec{D} \, d\vec{v} \quad — (2)$$

combining eq (1) and (2) we get

$$\int \nabla \cdot \vec{D} \, d\vec{v} = Q_{enclosed} \quad — (3)$$

volume charge density can be defined as-

$$\rho v = \frac{dQ}{dv}$$

$$dQ = \rho v dv$$

on integrating above eq^n we get

$$Q = \int \rho v dv \quad — (4)$$

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subtituting (4) in (3) we get

$$\int \nabla . D dv = \int \rho v dv$$

$$\boxed{\nabla . D dv = \rho v}$$

This is required 1st maxwell equation.

Maxwell Second Equation

Maxwell second equation is based on gauss law on magnetostatics.

Gauss law on magnetostatics states that "closed surface integral of magnetic flux density is always equal to total scalar magnetic flux enclosed within that surface of any shape or size lying in any medium."

$$\oint \vec{B} . ds = \phi enclosed \quad -(1)$$

Hence we can conclude that magnetic flux cannot be enclosed within a closed surface of any shape.

$$\oint \vec{B} . ds = 0 \quad -(2)$$

Applying the gauss divergence theorem to eqn (2) we can convert it into volume integral by taking the divergence of the same vector.

$$\oint \vec{B} . ds = \int \nabla . \vec{B} dv \quad -(3)$$

Subtituting eqn (3) in (2) we get -

$\int \nabla \cdot \vec{B} dv = 0$ — ④

Here to satisfy the above eqn either

$\int dv = 0$

or $\nabla \cdot \vec{B} = 0$

The volume of any body or object can never be zero

Thus we arrive at maxwell's second equation.

$$\boxed{\nabla \cdot \vec{B} = 0}$$

Where $\vec{B} = \mu\vec{H}$ is flux density

Maxwell Third Equation

Maxwell's 3rd eqn is derived from faraday's laws of Electromagnetic induction. it states that "Whenever there are n- turns of conducting coil in a closed path placed in a time-varying magnetic field, an alternating electromotive force gets induced in each coil."

Lenz's law gives this. which states, "An induced electromotive force always opposes the time-varying magnetic flux."

Mathematically it is expressed as-

Alternating emf,

$$emf_{alt} = -N\frac{d\phi}{dt} \quad — ①$$

N = no of turns in a coil.

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ϕ = scalar magnetic flux

Let $N = 1$

$$emf_{alt} = -\frac{d\phi}{dt} \quad — ②$$

here scalar magnetic flux can be replaced by-

$$\phi = \int \vec{B} \cdot \vec{ds} \quad — ③$$

Substitute eq^n ③ in ②.

$$emf_{alt} = -\frac{d}{dt} \int \vec{B} \cdot \vec{ds}$$

which is partial D.E. given by -

$$emf_{alt} = \int -\frac{d\vec{B}}{dt} \cdot \vec{ds} \quad — ④$$

The alternating electromotive force induced in a coil is basically a closed path.

$$emf_{alt} = \oint \vec{E} \cdot \vec{dl} \quad — ⑤$$

Substituting eq^n ⑤ in ④ we get -

$$\oint \vec{E} \vec{dl} = \int -\frac{\partial \vec{B}}{\partial t} \cdot ds \quad — ⑥$$

$$\oint \vec{E} \vec{dl} = \int (\nabla \times \vec{E}) \cdot \vec{ds} \quad — ⑦$$

(using stoke's theorem)

Substituting eq^n ⑦ in ⑥ we get

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$$\int (\nabla \times \vec{E}) \, d\vec{s} = \int -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{—} \; (8)$$

The surface integral can be canceled on both sides and we get

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

This is required 3rd maxwell's equation.

Maxwell's Fourth Equation.

Maxwell's fourth equation is derived from Ampere's law which states that " Magnetic field can be either produced by electric current or by the altering electric field". The magnetic field vector's closed line integral is equal to the total quantity of scalar electric field present in the path of that shape. This Maxwell's eqn also defines the displacement current. The electric current and displacement current through a closed surface is directly proportional to the induced magnetic field around any closed loop.

Maxwell added the displacement current to Ampere's law. Mathematical representation of maxwell's equation Fourth Equation.

Closed line integral of magnetic field vector = Total quantity of scalar electric field present,

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$$\oint \vec{H}\cdot\vec{dl} = I \quad -①$$

$$\oint \vec{H}\cdot\vec{dl} = \int (\nabla\times\vec{H})\cdot ds \quad -②$$

(by stoke's theorem)

using ② in eq ①

$$\int (\nabla\times\vec{H})\cdot\vec{dl} = I \quad -③$$

$\int (\nabla\times\vec{H})\cdot\vec{dl}$ = Vector quantity.

I = scalar quantity.

Multiply I by density vector,

$$\vec{J} = \frac{I}{s}\hat{a}N$$

$\vec{J}$ = Difference in scalar electric field / Difference in vector electric field $\vec{J}$.

$$\frac{dI}{ds.dl} = \vec{J}\cdot ds$$

$$I = \int \vec{J}\cdot\vec{ds} \quad -④$$

Using eq ④ in eq ③.

$$\int (\nabla\times\vec{H})\cdot\vec{dl} = \int \vec{J}\cdot\vec{ds} \quad -⑤$$

Cancelling the surface integral from both sides, we get maxwell's fourth equation

$$\boxed{\vec{J} = \nabla\times H}$$

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$$\boxed{(\nabla \times \vec{H}) = \vec{J} + \vec{J}_D}$$

Ques 3 State and prove poynting theorem.

Ans:-

Statement :- This Theorem states that the cross product of electric field vector, E and magnetic field vector, H at any point is a measure of the rate of flow of electromagnetic energy per unit area at that point, that is

$$P = E \times H$$

P = Poynting vector , $P \perp E$ and H

Proof :-

Consider maxwell's fourth eq^n that is

$$\text{del} \times H = J + \varepsilon \frac{dE}{dt}$$

or $J = (\text{del} \times H) - \varepsilon \frac{dE}{dt}$

$$E \cdot J = E \cdot (\text{del} \times H) - \varepsilon E \cdot \frac{dE}{dt} \quad \text{— (1)}$$

(taking dot product with E)

use vector identity

$$\text{del} \cdot (E \times H) = H \cdot (\text{del} \times E) - E \cdot (\text{del} \times H)$$

or $E \cdot (\text{del} \times H) = H \cdot (\text{del} \times E) - \text{del}\,(E \times H)$

By substituting value of $E.(del \times H)$ in eq^n 1 we get

$$E.J = H.(del \times E) - del.(E \times H) - \varepsilon E \frac{dE}{dt} \quad -(2)$$

also from Maxwell's third eq^n (Faraday's law of electromagnetic induction).

$$del \times E = \mu \frac{dH}{dt}$$

by substituting value of $del \times E$ in eq^n 2 we get

$$E.J = \mu H.\frac{(dH)}{dt} - \varepsilon E.\frac{dE}{dt} - del.(E \times H) \quad -(3)$$

we can write

$$H.\frac{dH}{dt} = \frac{1}{2}\frac{dH^2}{dt} \quad -(4a)$$

$$E.\frac{dE}{dt} = \frac{1}{2}\frac{dE^2}{dt} \quad -(4b)$$

By substituting eq^n 4a and 4b in eq^n 3 we get.

$$E.J. = -\frac{\mu}{2}\frac{dH^2}{dt} - \frac{\varepsilon}{2}\frac{dE^2}{dt} - del.(E \times H)$$

$$E.J = -\frac{d}{dt}\left(\frac{dH^2}{2} + \frac{\varepsilon E^2}{2}\right) - del.(E \times H)$$

By taking volume integral on both sides we get.

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$$\int E.J.\,dv = \frac{-d}{dt}\int\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)dv - \int del.(E\times H)\,dV \quad -(5)$$

apply gauss's divergence theorem to second term of R.H.S. to change volume integral into surface integral, that is

$$\int del.(E\times H)\,dV = \int (E\times H).ds$$

Substitute above eq^n in eq^n (5)
Thus

$$\int E.J.\,dv = \frac{-d}{dt}\int\left[\varepsilon E^2/2 + \mu H^2/2\right]dv - \int (E\times H).ds \quad -(6)$$

or $$\int (E\times H).ds = \int -\frac{d}{dt}\left[\varepsilon E^2/2 + \mu H^2/2\right]dv - \int E.J.\,dV$$

$$\boxed{\int (E\times H).ds = \int -\frac{d}{dt}\left[\varepsilon E^2/2 + \mu H^2/2\right]dv - \int E.J.\,dV}$$

Hence proved